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NOTE

A Note on a Continuous Approximate Selection Theorem

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The purpose of this note is to show a new generalization of the continuous approximate selection theorem of F. Deutsch and P. Kenderov (1983, *SIAM J. Math. Anal.* **14**, 185–194).

Let X be a subset of a Hausdorff topological space E, Y be a locally convex topological vector space, and $\mathcal{O}(x)$ be denoted the collection of all neighbourhoods of x. Suppose that $F: X \to 2^Y$ is a multivalued mapping with nonempty values and θ denotes the origin point of Y. F is called almost lower semicontinuous (abbreviated as a.l.s.c.) at $x \in X$ if for each $V \in \mathcal{O}(\theta)$ there exists an $U \in \mathcal{O}(x)$ such that $\bigcap \{F(u)+V : u \in U\} \neq \emptyset$. We say that F has continuous approximate selections if for every $V \in \mathcal{O}(\theta)$ there is a continuous function $f: X \to Y$ satisfied that $f(x) \in F(x)+V$ for all $x \in X$.

THEOREM A. *If X is paracompact and $F(x)$ is nonempty convex for each $x \in X$, then F is a.l.s.c. if and only if F has continuous approximate selections.*

Proof. Necessity. It is analogous to the proof of Theorem 2.4 of [1], but let the convex neighborhood of the origin of Y be V instead of the ball $B_\varepsilon(\theta)$.

Sufficiency. For any $V \in \mathcal{O}(\theta)$ there is a balanced neighborhood $W \in \mathcal{O}(\theta)$ such that $W+W \subset V$. Let $f: X \to Y$ be a continuous mapping such that $f(x) \in F(x)+W$ for all $x \in X$. By virtue of the continuity of f, there exists for each $x \in X$ a neighborhood $U \in \mathcal{O}(x)$ such that $f(u) \in f(x)+W$ for all $u \in U$. From the balancedness of W, we have that $f(x) \in f(u)+W$ for all $u \in U$. Consequently,

$$f(x) \in f(u)+W+W \subset F(u)+V \qquad \forall u \in U.$$

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I.e., $\bigcap \{F(u)+V : u \in U\} \neq \emptyset$. Hence F is a.l.s.c. and the proof is complete. ∎

If Y is a normed linear space, then Theorem A is due to Deutsch and Kenderov [1, Theorem 2.4]. Furthermore, if F is strengthened to a lower semicontinuous mapping, then Theorem A is due to Michael [2, Lemma 4.1].

We like to point out that the proof of Theorem A does not depend on any metric topology. By the way, if $X = Y$ in Theorem A, then F has an almost fixed point. Using this result, we can study approximate methods for the equilibrium point of abstract economies (or generalized games).

REFERENCES

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